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Overreact Analysis in the American Stock Market: A Fuzzy C-means Algorithm Approach

Renato Aparecido Aguiar and Roberto Moura Sales

Abstract— **In this paper, empirical tests, based on the fuzzy clustering means algorithm for the analysis of overreaction and underreaction hypothesis in the American stock market are presented. Such methodology is strongly connected with two heuristics of behavioral finance theory: representativeness heuristic and anchoring heuristic. The proposed methodology is used to form portfolios through financial ratios of public companies and the results obtained are consistent with the strong influence of overreaction in the American stock market. The analysis is applied for stocks from oil and gas, textile and, steel and iron sectors, with financial indexes ranging from 1999 to 2007.**

Index Terms - **Behavioral Finance, Fuzzy Clustering Means, Overreaction, Underreaction.**

I. Introduction

The development of methodologies to overreaction and underreaction analysis in the financial market has been object of intense research. Generally speaking, financial decisions aim at maximizing future profit and, in this context, the evolution of the basic premises has given birth to the appearance of clashing theories as, for instance, the theory of efficient markets and the theory of behavioral finance [11]. Each of these theories has been structured with basis on the fundamental contributions of a large number of authors. Nonetheless, the fact that each one of them contains in its origins works which have been awarded the Nobel Prize for Economics, such being the case of the portfolio theory proposed by H.M. Markowitz [8], and the theory of behavioral finance proposed by D. Kahneman and V. L. Smith [5], is undeniably of the utmost relevance. This is enough to give a clear idea of the importance of decision making in the economic theory. Summing up, the Theory of the Efficient Market assumes that the investor is rational, therefore likely to avoid risk-taking. As a consequence of this hypothesis, investors' decisions are made with basis on information strongly supported by statistics and probabilities which supposedly project the future performance of the assets as well as of the financial market as a whole. Differently, the Behavioral Theory uses models which take into account factors of psychological nature which tend to influence the investors in decision making; in other words, the investors are not totally rational and their decisions are affected by their preferences and beliefs, associated to heuristics and rules of thumb. Despite the clash of opinions which have promoted an intense debate involving these two theories, the behavioral finance has proved to be an adequate tool when tackling several problems. In the area of asset pricing, for instance, it has been utilized to interpret phenomena involving the return of assets, such as overreaction and underreaction from the market to news. One has learned a lot about the behavior of investors and analysts from the analysis of information about the behavior of the financial market. In corporate finance, the behavioral approach has been calling attention to attitudes such as the excessive aversion to risk-taking as well as to excessive optimism.

The target of the present paper is the study of phenomena of overreaction and underreaction in what concerns the American stock market. Some of the papers which focus on similar problems in the international stock markets are: Reference [3] for the American market, Reference [6] for the Chinese market and Reference [7] for the Brazilian market. Reference [1] examines conditions that lead to overreaction and underereaction in analysts' earnings forecasts. These models have evidenced anomalies (overreaction/underreaction) in the stock market.

The bond still hardly investigated in the literature, as observed in [9], between the theory of fuzzy sets, originally proposed by L. A. Zadeh in 1965 [12], and the heuristics of the theory of behavioral finance is explored for the developments here presented. A methodology is developed for the rating of a set of stocks in the American market with basis on financial indexes of the corresponding firms. It is thus demonstrated that the proposed methodology incorporates heuristic influences from the behavioral finance theory. Empirical tests for the overreaction and underreaction hypothesis are presented using data from 1994 to 2007 in the American market.

The importance of phenomena of overreaction and underreaction lies in the fact that in decision making overreaction justifies the option for the use of the strategy named contrarian strategy while underreaction tends to justify the choice of the strategy of momentum.

II. Mathematical Basis

The fuzzy set theory possesses as one of its main characteristics the fact of allowing the treatment of linguistic variables, such as hot, very hot, high, low, advisable, not advisable, highly risky, etc.

A fuzzy set is a set containing elements that have varying membership degrees in the set and such elements are mapped

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Renato Aparecido Aguiar is with the Centro Universitário da FEI, Dept of Electrical Engineering (e-mail: preraguiar@fei.edu.br).
Roberto Moura Sales is with the Escola Politécnica da USP, Dept of Electrical Engineering (e-mail: roberto@lac.usp.br).

to a universe of membership values using a function-theoretic form, which maps a universe of objects X onto the unit interval [0,1]. If an element x is a member of fuzzy set A, then this mapping is given by $\mu_A(x) \in [0,1]$ [14]. According original idea of Zadeh [12], a fuzzy set of the universe of objects X is defined as a function μ_A that maps X into [0,1], that is, $X \to [0,1]$.

The resulting property when considering linguistic variables to characterize objects is that, instead of belonging or not to a certain set, as stated by the classic set theory, these objects will have pertinence indexes associated with different sets. A detailed presentation of the main concepts of the fuzzy theory can be found in [12] and [13].

Definition 1: Let the set $X = \{x_1, x_2, ..., x_m\}$, $C_1, C_2, ..., C_n$ subsets of X and real numbers $0 \le \mu_i(x_j) \le 1, i = 1,2,...n$, $j = 1,2,...,m$, such that, for every $j = 1,2,...,m$, one has $\sum_{i=1}^{n} \mu_i(x_j) = 1$. Under these conditions, $\mu_i(x_j)$ is denoted membership degree of the element x_j with respect to fuzzy subset C_i. The membership degree may be understood as a measure of the degree of affinity, similarity or compatibility among elements.

In the applications the Fuzzy Theory has been employed in a number of areas such as Engineering [15], Medicine [16], as well as Economics [17].

Among the techniques for the grouping or classification of elements in subsets of a given set, the Fuzzy Clustering Means – FCM algorithm has been proved to be an effective tool in those cases in which the features or attributes of the analyzed elements can be represented by a vector of real numbers. In such cases, the FCM algorithm allows identifying clusters of elements from a matrix of dimension $n \times p$, being n the number of elements and p the dimension of the vectors of features of these elements [13]. As in the specific application of this paper the analyzed elements are grouped in 2 subsets only, the presentation is specified for this case. Thus, let $x_1, x_2, ..., x_m$ elements of X and consider the problem of grouping these elements in 2 subsets C_1 e C_2. The FCM algorithm determines the subsets C_1 and C_2 via the solution of the following problem.

Given the elements $x_1, x_2, ..., x_m$, described as vectors of dimension p, determine the vectors c_1 and c_2, also of dimension p, and $\mu_1(x_j) \ge 0$ and $\mu_2(x_j) \ge 0$, $j = 1,2,...,m$, such that $\mu_1(x_j) + \mu_2(x_j) = 1,\ j = 1,2,...,m$ and the function $\sum_{i=1}^{2}\sum_{j=1}^{m}[\mu_i(x_j)^2 \|x_j - c_i\|^2]$ is minimized.

The solution of such optimization problem is given by [2]:

$$c_i = \frac{1}{\sum_{j=1}^{m} (\mu_i(x_j))^2} \sum_{j=1}^{m} (\mu_i(x_j))^2 x_j \quad \mathrm{i} = 1,2 \tag{1}$$

$$\mu_i(x_j) = \frac{\dfrac{1}{\|x_j \quad c_i\|^2}}{\sum_{k=1}^{m} \|x_k - c_i\|^2} \quad \mathrm{i} = 1,2 \quad \mathrm{j} = 1,2,...,\mathrm{m} \tag{2}$$

Vectors c_i are called centers. If $\mu_1(x_i) > \mu_2(x_i)$ one says that the element x_i is associated to C_1 and if $\mu_2(x_j) > \mu_1(x_j)$ then x_j is associated to C_2.

As one can well observe, the calculation of c_i, through (1), depends on $\mu_i(x_j)$. These, on their turn, depend on c_i, according to (2). The solution can be obtained iteratively, by the algorithm named FCM, whose steps are next described.

Step 1: Initiate with membership degrees, such that $\mu_1(x_j) + \mu_2(x_j) = 1$, $j = 1,2,...,m$ and $\mu_1(x_j) \ge 0$ and $\mu_2(x_j) \ge 0$, $j = 1,2,...,m$;

Step 2: Calculate the centers c_1 and c_2, by (1);

Step 3: Recalculate the new membership degrees, via (2), by utilizing the centers obtained in step 2.

Repeat steps 2 and 3 until the objective function do not decrease, according to the assumed precision. In order to achieve the global minimum of square-error, different partitions must be chosen such that the final partition results always the same [12].

III. BEHAVIORAL FINANCE

In the beginning of the 70s, time when the Theory of Efficient Market had attained a high degree of influence in decision making in Economics, the search for the comprehension of abnormalities of behavior in financial markets world over brought about the perspective of the inclusion of concepts of Psychology and Sociology in the economic analysis, to such an extent that Kaheneman's paper [5], and Smith [18], deserved to be awarded the Nobel Prize of Economics in 2002. The elements involved in this new approach led to the elaboration of the theory named Theory of Behavioral Finance.

According to the behavioral theory, individuals make decisions guided by heuristics, or practical rules, thinking in a way which deviates from the statistic rules.

Several heuristics influence decision making. In particular, the heuristics of representativeness and anchoring [9], are directly related to the theory of fuzzy sets, in the way it is seen in this paper.

Briefly speaking, the heuristic of representativeness is associated with the similarity between the considered elements. In the sequence, is presented a classical example, in which decision making is strongly influenced by descriptive pieces of information, even when the probability of occurrence is known, that is, individuals prefer to value descriptive information instead of considering the probability of occurrence of the fact.

In this classical example of the literature some individuals are asked to guess what the occupation of a person chosen randomly in a group of ten people, knowing that eight people in the group are truck drivers and two others are accountants. In the first case the ten people are dressed in the same manner and, after one of these ten having been chosen among them, the majority of the participants, relying on the known probability, decided that this person would be a truck driver. In the second case, however, an element of ambiguity was added, that is, the ten people were differently dressed, and an individual wearing a suit, glasses and carrying a briefcase was chosen. In this case, the majority of participants identified this person as being an accountant, even though the probability of this individual being a truck driver was greater than the probability known beforehand of his being a broker [27].

In this example, the man wearing a suit, glasses and carrying a briefcase has more similarity with the set of accountants and, therefore, less similarity with the set of truck drivers Experiments of this kind strengthen the hypothesis that a method which makes use of fuzzy sets is more adequate to shape decision making under conditions of ambiguity than statistic methods [10].

In the context of decision making in Economics, individuals under the influence of the heuristic of representativeness tend to produce extreme predictions, or overreaction [1], in which former losers tend to be winners in the future and vice-versa [19]. In other words, the stock market overreaction maintains that a given stock decreases (increases) too far in price as a consequence of recent bad (good) news associated with the stock. Thus, traders who are not sure about the intrinsic value of a stock will be too optimistic about its value when the firm is winning and too pessimistic when it is losing [20]. In the fuzzy set theory the similarity is directly related to the membership degree, which is a more suitable tool to describe such characteristic than statistic methods [10].

The heuristic of anchoring establishes that people often base themselves on elements or conditions of reference in order to make decisions. The heuristic of anchoring, differently from the heuristic of representativeness, leads to excessive moderation in decision making, thus causing the underreaction phenomenon, in which former winners tend to be future winners, and former losers tend to be future losers [19]. The heuristic of anchoring is associated with conservative decision making, causing people to resist quick changes in their beliefs in the presence of new information. An experiment conducted in [27] shows the influence of the anchoring heuristic in the decision of an individual. In this experiment, two student groups must estimate the value of an expression in five seconds:

$8\times7\times6\times5\times4\times3\times2\times1$ for group 1 and

$1\times2\times3\times4\times5\times6\times7\times8$ for group 2.

Although the correct answer to the two sequences is 40.320, the average estimate obtained for the sequence 2 was 512, while the average estimate obtained by the group 1 (descending sequence) was 2.250. This occurs because in the descending sequence the first steps of multiplication (from left to right) produce a number greater than in the case of the ascending sequence.

In terms of the theory of fuzzy sets, a decision based on this heuristic is focused on the element of stronger reference in the set, that is, the element of total membership $\mu(x)=1$ [9].

According to what has been mentioned formerly, the Theory of Behavioral Finance incorporates elements which are different from those considered in the Theory of Efficient Market [21]. This fact has allowed the development of some new models in order to explain possible abnormalities in the financial market, and such abnormalities when inexplicable with the help of classical tools has found satisfactory explanations in the Theory of Behavioral Finance [22]. Some models related to the heuristics of representativeness and anchorage are:

A model of the investor's sentiment has been proposed in [23]. This model is based on behavioral deviations as a consequence of the already mentioned heuristics, leading investor to either underreact or overreact to the available information.

Daniel, Hirshleifer and Subrahmanyam's model [24], aims at conciliating the empirical facts of overreaction and underreaction. According to these authors, the investors with no information do not present behavioral deviations, while the investors who possess information are influenced by two factors of deviation: overconfidence, which is associated to the heuristic of representativeness and excessive value attributed to their perceptions, which is associated with the heuristic of anchoring.

In [3], based on the accumulation of monthly returns in the American stock market, evidences of overreaction associated to the heuristic of representativeness in the prices of assets are presented.

To strengthen the overreaction and underreaction hypotheses in the stock market, in the next section a new methodology for analysis of overreaction and underreaction is presented.

IV Methodology for Empirical Tests of Underreaction and Overreaction

In this section, the methodology employed for the development of a new analysis of overreaction and underreaction is introduced. This methodology comprehends two steps: pattern recognition and stock rating. The data or features of the stocks utilized by the model are financial indexes of open firms, including some return indexes related to stock evaluation, profitability and debt. These indexes have been collected every trimester from the Economatica data base [4], between the 4th trimester/1994 and the 3rd trimester/2007.

The relation between these ratios and the financial return of the stocks is a theme which is highly discussed in the literature [25]. For the development of this paper several sets of financial ratios related to marketability, profitability, indebtedness and stocks rating have been tested, grouped in different ways. The ratios here selected and effectively adopted, which have produced the best results, are divided into:

- Profitability ratios: net profit margin, return on net worth, return on assets [26];
- Debt ratio: debt to equity [26];
- Market indicators: price on book value of equity, price on earning per share [25].

The two steps of the proposed model are next described.

Step 1: in this step, named pattern recognition, the Fuzzy Clustering Means algorithm classifies the stocks of a given group. This analysis has been based on data in the period between the 4th trimester/1999 and the 3rd trimester/2004. In each trimester *t*, the FCM algorithm was applied to the pattern matrix $n \times p$, in which each line corresponds to one firm of the group and each column corresponds to the financial indexes associated to the firms. Two clusters have been obtained and the average financial return that each cluster produces at the end of the trimester $t+1$ is calculated according to (3). In the step 1 of the FCM algorithm was used to get the winner centers and the loser centers in each trimester.

$$r_{t+1} = \frac{1}{n}\sum_{i=1}^{n} \ln \frac{P_{t+1}^{i}}{P_t^{i}} \tag{3}$$

where P_t^i is the value of stock *i* at the end of trimester *t*, P_{t+1}^i is the value of the stock *i* at the end of period $t+1$ and *n* is the number of stocks classified.

The cluster with larger average financial return is called winner cluster and the one with smaller average financial return is called loser cluster. Each group is represented by vector of financial ratios calculated iteratively using (2) and (3) until the objective function is minimized. In this sense, according to the definition of the anchoring heuristic, in which the estimates are formed from an initial value to produce the final answer, the determination of these vectors of financial ratios may be seen as based on the anchoring heuristic. In this step, the classification of the groups as winner or loser has been possible only at the end of the trimester $t+1$, that is, the classification is a posteriori.

Step 2: the aim of this step is to classify, at the end of trimester t, the cluster of stocks with performance supposedly winner or loser at the end of trimester $t+1$. Thus, differently from step 1, the aim is to set a classification a priori.

For this second step the clusters formed by the centers of the 1st trimesters, 2nd trimesters, 3rd trimesters and 4th trimesters from 1999 to 2004 are considered separately. Then, the FCM algorithm is applied again in order to identify a winner center and a loser center for each set of 1st trimesters, 2nd trimesters, 3rd trimesters and 4th trimesters. In the classification of a stock at the beginning of a particular trimester, it is enough to calculate the membership degrees related to the winner and the loser centers corresponding to that trimester. For each trimester, the group of promising stocks will be called winner portfolio and the group of non-promising stocks will be called loser portfolio. For the numerical results, the classification of stocks was made in the period between the 4th trimester/2004 and the 4th trimester/2007.

The assets are classified in each group with a degree of similarity which is calculated by equation 3. The degree of similarity is so much greater the shorter the distance between the center vector and the ratios vector of the asset. In this sense, there is an evident link between the proposed methodology for classification and the representativeness heuristic, which is based on the degree of similarity that an object A belongs to class B.

The phenomena of overreaction and underreaction have been largely investigated through empirical research [3], [6]. The procedure for the empirical tests of such hypothesis along with the tests of statistic significance presented in this paper are similar to those presented in [3].

Firstly, by using the winner and the loser centers, stocks are classified. Thus, winner and loser portfolios for each trimester are formed. Next, the corresponding residual return is calculated for each week of the trimester $t+1$, according to (4), (5) and (6), for the winner portfolio.

$$RR_{t+1,j}^{W} = r_{t+1,j}^{W} - r_{t+1,j}^{M} \tag{4}$$

$$r_{t+1} = \frac{1}{n}\sum_{i=1}^{n} \ln \frac{P_{t+1}^{i}}{P_t^{i}} \tag{5}$$

$$r_{t+1,j}^{IM} = \ln \frac{IM_{t+1,j}}{IM_t} \tag{6}$$

where $RR_{t+1,j}^{W}$ is the residual return for the portfolio in the week j of the trimester $t+1$, $r_{t+1,j}^{W}$ is the return for the portfolio in the week j of the trimester $t+1$, $r_{t+1,j}^{M}$ is the return associated with the market index in the week *j* of the trimester $t+1$, $P_{t+1,j}^{i}$ is the value of the stock *i*, of the portfolio at the end of the week j of the trimester $t+1$, P_t^i is the value of the stock i of the portfolio at the end of trimester t, $IM_{t+1,j}$ is the market index at the end of week *j* of the trimester $t+1$, IM_t is the market index at the end of trimester *t* and *n* is the number of stocks in the portfolio. Similar calculations are performed for the loser portfolio.

From the residual returns corresponding to the weeks of each trimester, the average residual returns of the winner portfolio ARR_t^W and of the loser portfolio ARR_t^L are calculated from the 1st trimester/2005 up to the 4th trimester/2007. The hypothesis of overreaction says that $ARR_t^W - ARR_t^L < 0$ [7]. In order to evaluate whether the difference between average residual returns in each trimester is meaningful, a test of statistic-t is performed. The null hypothesis to be tested is $H_0 : ARR_t^L - ARR_t^W = 0$, against the alternative hypothesis of overreaction $H_A : ARR_t^W - ARR_t^L < 0$. Similarly, the alternative hypothesis for underreaction is $H_A : ARR_t^W - ARR_t^L > 0$.

Furthermore, are calculated, according to (7) and (8), the cumulative ARR_t^W $\left(CARR_t^W\right)$ for the winner portfolio and cumulative ARR_t^L $\left(CARR_t^L\right)$ for the loser portfolio from 1st trimesters of 2005 to 4th trimesters of 2007 for investigate the influences of overreaction and underreaction during the period considered.

$$CARR_t^W = \sum_t ARR_t^W \tag{7}$$

$$CARR_t^L = \sum_t ARR_t^L \tag{8}$$

V. The American Stock Market – A Case Study

In this section the methodology proposed in section IV is utilized to form portfolios and, next, tests for the hypothesis of overreaction and underreaction are performed.

The market index adopted is the S&P 500 index and the data from stocks of the oil and gas sector, textile sector and steel and iron sector are considered in the period ranging from 1999 to 2007. In step 1 of the algorithm data from the 4th trimester/1999 to the 3rd trimester/2004 are used to get the winner and the loser centers and the step 2 data from the 4th trimester/2004 to the 4th trimester/2007 are used to form winner and loser portfolios.

As an example of the obtained results, in Fig.1 the graphs of the residual returns for the winner portfolio (W) and for the loser portfolio (L) obtained at the end of the 1st trimester of 2007, as well as the difference between them during each week of the 2nd trimester of 2007 for the oil and gas sector are presented.

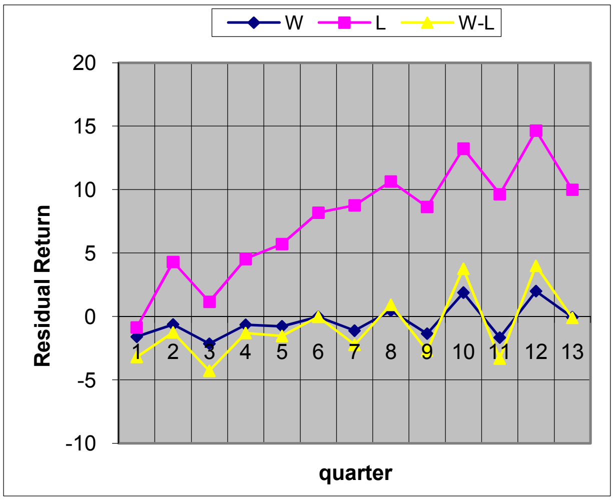

Figure 1. Weekly Residual Returns – Oil and Gás Sector

In this case, evidences of overreaction are obvious. By calculating the average residual returns for the winner and loser portfolios for this quarter, one can observe that the loser portfolio outperform in approximately 8.06% the average residual return of the winner portfolio, meaningful at the level of 5% (statistic t:-7.989).

Table I shows the results of this analysis for every quarter of 2005, 2006 and 2007 for stocks of the oil and gas sector. One can notice that in the majority of quarters there are statistically meaningful evidences of overreaction. Furthermore, the $CARR_t^L = 26{,}464$ from loser portfolio outperform the $CARR_t^W = 10{,}975$ from winner portfolio in 15,489%, after twelve quarters (three years), supporting the overreaction hypothesis for the oil and gas sector.

In table II and table III are presented the results of this procedure for all trimesters from 2005 to 2007 in the case of stocks of textile and steel and iron sectors, respectively.

TABLE I. Average Residual Returns and Test-t for the Petrol/Petrochemical Sector Statistically Meaningful at the Level of $^{(*)}$5% and $^{(**)}$10%.

Trimester/ year	$ARR_t^V - ARR_t^P$	Test-t
1st trim/2005	9,266	4,058*
2nd trim/2005	2,112	0,462
3rd trim/2005	-3,968	-1,922**
4th trim/2005	4,594	2,552*
1st trim/2006	-5,970	-2,580*
2nd trim/2006	-0,338	-0,438
3rd trim/2006	0,705	0,742
4th trim/2006	-6,083	-5,148*
1st trim/2007	-5,820	-2,525*
2nd trim/2007	-8,455	-7,989*
3rd trim/2007	0,027	0,021
4th trim/2007	-1,557	-1,654**

TABLE II. Average Residual Returns and Test-t for the Textile Sector Statistically Meaningful at the Level of $^{(*)}$5% and $^{(**)}$10%.

Trimester/ year	$ARR_t^V - ARR_t^P$	Test-t
1st trim/2005	3,189	1,150
2nd trim/2005	-1,586	-0,682
3rd trim/2005	-3,930	-4,645*
4th trim/2005	7,533	2,437*
1st trim/2006	2,698	1,039
2nd trim/2006	0,219	0,122
3rd trim/2006	-5,446	-2,488*
4th trim/2006	-2,591	-0,722
1st trim/2007	4,054	1,825**
2nd trim/2007	9,298	2,222**
3rd trim/2007	-6,198	-1,096
4th trim/2007	-14,037	-2,693*

TABLE III. Average Residual Returns and Test-t for the Steel and Iron Sector Statistically Meaningful at the Level of $^{(*)}$5% and $^{(**)}$10%.

Trimester/ year	$ARR_t^V - ARR_t^P$	Test-t
1st trim/2005	-0,412	-0,090
2nd trim/2005	-30,580	-1,954**
3rd trim/2005	2,867	0,501
4th trim/2005	-6,596	-1,595
1st trim/2006	-4,009	-0,530

2nd trim/2006	-18,789	-4,025*
3rd trim/2006	6,746	2,542*
4th trim/2006	5,905	1,367
1st trim/2007	-2,834	-0,640
2nd trim/2007	24,534	2,369*
3rd trim/2007	5,661	2,423*
4th trim/2007	-10,368	-2,654*

The same procedure has been adopted for stocks in the textile sector and steel and iron sector and the obtained results are presented in table II and table III, respectively. In this case, fluctuations between overreaction and underreaction, statistically meaningful, are observed. Though, the $CARR_t^L = 48{,}664$ from loser portfolio outperform the $CARR_t^W = 20{,}789$ from winner portfolio in 27.875% for the iron and steel sector and, $CARR_t^L = 2{,}180$ from loser portfolio outperform the $CARR_t^W = -4{,}637$ from winner portfolio in 6.817% for the textile sector after twelve quarters (three years), indicating a strong influence of overreaction in steel and iron sector and textile sector.

VI. Conclusion

In this paper a methodology based in the Fuzzy Clustering Means Algorithm for stocks rating is proposed and a procedure for empirical tests of overreaction and underreaction in the stock market is presented .

The proposed methodology is based on the Fuzzy Sets Theory, which on its turn is closely related to the Theory of Behavioral Finance. To form the portfolios, the proposed methodology utilizes financial ratios of public companies.

Numerical results are presented when applying the proposed methodology to the American stock market. Three sets of stocks are considered in this case: stocks of the oil and gas sector, stocks of the steel and iron sector and stocks of the textile sector. The oil and gas sector presents statistically meaningful evidences of overreaction while the portfolios formed by stocks of the textile sector and stocks of the steel and iron sector present fluctuations between overreaction and underreaction, statistically meaningful. Therefore, after twelve quarters (three years), the loser portfolio results in an abnormal profit with regard to winner portfolio for all the sectors considered. The results obtained are consistent with the overreaction hypothesis in the American stock market and confirm the findings in [3].

These facts, added to the existing bond between fuzzy sets and the Theory of Behavioral Finance point, thus, to the influence of the heuristic of representativeness in the behavior of the American stock market and it is consistent with the strong influence of the overreaction in the market.

The importance of phenomenon of overreaction and underreaction lies in the fact that in decision making overreaction justifies the option for the use of the strategy named contrarian strategy while underreaction tends to justify the choice of the strategy of momentum. In the case of the momentum strategy, past winners are bought and past losers are shorted and, in the contrarian strategy, past winners are shorted and past losers are bought.

The results obtained suggests thus that abnormal profits could be obtained with a systematic application of the contrarian strategy in the considered American stock market and shows that an investor could obtain abnormal profits with the short of the winner portfolio and the buy of the loser portfolio.

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